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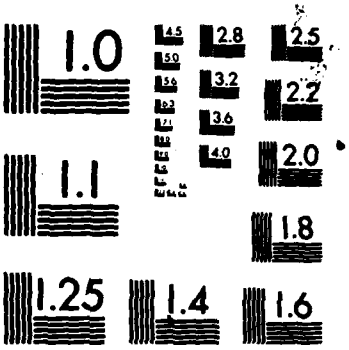

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Department of Mathematics, Statistics and
Computer Science
University of Illinois at Chicago
Technical Report No. 85-07
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ABSTRACT

The class of new better than used in expectation life distributions is shown to be closed under the formation of parallel systems with independent and identically distributed components. The class of differentiable life distributions with decreasing mean residual life is also proved to have the same closure property.

1. INTRODUCTION.

The class of new better than used in expectation (NBUE) life distributions arises quite naturally in considering maintenance and replacement policies. The class of life distributions with decreasing mean residual life (DMRL) is receiving a good deal of current attention, especially in biostatistics. It is well known that the NBUE class and the DMRL class are not closed under the formation of general coherent systems with independent components. In fact a series system with independent and identically distributed (i.i.d.) NBUE components may not have an NBUE life distribution, (see Barlow and Proschan (1981), page 183).

However, the question as to whether or not the NBUE and the DMRL properties are preserved under the formation of parallel systems with i.i.d. units has remained open (see Klefsjo (1985)). In this note we show that the class of NBUE life distributions and the class of differentiable DMRL life distributions are both closed under the formation of parallel systems with i.i.d. units.

Keywords
random variables; distribution functions

2. THE RESULTS.

Recall that a nonnegative random variable T (or equivalently its distribution F) is said to have the NBUE property, or simply to be NBUE if

$$\int_t^\infty \bar{F}(u)du \le \mu\bar{F}(t), \tag{2.1}$$

for all $t \ge 0$, where $\bar{F}(t) = 1 - F(t)$ and $\mu = ET < \infty$.

Similarly a nonnegative random variable T (or equivalently its distribution F) is said to be DMRL if $F(0) = 0$ and

$$\int_t^\infty \bar{F}(u)du/\bar{F}(t) \quad \text{decreases in} \quad t, \tag{2.2}$$

for all $t \geq 0$ for which $\bar{F}(t) > 0$, where again $ET < \infty$.

In the following theorem we show that the class of NBUE life distributions is closed under the formation of parallel systems with i.i.d. units.

Theorem 2.1. Let $T_1,\ldots,T_n$ be i.i.d. NBUE random variables with common distribution F. Let $T = \max(T_1,\ldots,T_n)$. Then T is NBUE.

Proof: Since $\mu = \int_0^t (1 - F(u))du + \int_t^\infty (1 - F(u))du$, condition (2.1) is equivalent to

$$\int_t^\infty [F(t)(1 - F(u))]/(1 - F(t))du \leq \int_0^t (1 - F(u))du, \tag{2.3}$$

where, to avoid trivialities, we tacitly assume that $1 - F(t) > 0$. We need to show that (2.3) remains true with $F^n(\cdot)$ replacing $F(\cdot)$, where $F^n(t)$ is the distribution function of T. To this end we only need to show that

$$\int_t^\infty F(t)(1 - F(u))/(1 - F(t))du \geq \int_t^\infty F^n(t)(1 - F^n(u))/(1 - F^n(t))du.$$

This last inequality follows readily by observing that

$$F(t)(1-F(u))/(1-F(t)) - F^n(t)(1-F^n(u))/(1-F^n(t)) =$$

$$= F(t)(1-F(u))/(1-F(t)) \times$$

$$\times \{1 - F^{n-1}(t)[1+F(u)+\ldots+F^{n-1}(u)]/[1+F(t)+\ldots+F^{n-1}(t)]\} \geq 0,$$

and the proof is now complete. □

In the following theorem we prove that the class of differentiable DMRL life distributions is closed under the formation of parallel systems with i.i.d. components. In view of the fact that a DMRL distribution is necessarily continuous, the added assumption of differentiability should not be too restrictive.

Theorem 2.2. Let $T_1,\ldots,T_n$ be i.i.d. DMRL random variables with common differentiable distribution function F. Let $T = \max(T_1,\ldots,T_n)$. Then T is DMRL.

Proof: Let f be the derivative of F. Then condition (2.2) is equivalent to

$$\int_t^\infty f(t)(1-F(u))/(1-F(t))^2\,du \leq 1. \qquad (2.4)$$

where again $1-F(t)$ is tacitly assumed to be positive. We now need to show that (2.4) remains true with $F^n(\cdot)$ and its derivative replacing $F(\cdot)$ and its derivative respectively. To this end it suffices to show that

$$\int_t^\infty f(t)(1-F(u))/(1-F(t))^2\,du \geq \int_t^\infty nF^{n-1}(t)f(t)(1-F^n(u))/(1-F^n(t))^2 du$$

Simple algebraic manipulations show that the above inequality

follows readily by establishing the following:

$$nF^{n-1}(t)[1 + F(u) + \ldots + F^{n-1}(u)] \leq [1 + F(t) + \ldots + F^{n-1}(t)]^2. \quad (2.5)$$

Now observe that $1 + F(t) + \ldots + F^{n-1}(t) - nF^{(n-1)/2}(t)$

$$= \sum_{j=0}^{[(n-1)/2]} [1 - F^{[(n-1)/2]-j}(t)][F^j(t) - F^{(n-1)/2}(t)] \geq 0, \text{ where}$$

$[(n-1)/2]$ denotes the largest integer less than or equal to $(n-1)/2$. This establishes (2.5) and the proof is now complete. □

Remark 2.3. The assumption of differentiability in Theorem 2.2 can be replaced by the assumption of absolute continuity. In this case derivatives need only exist almost everywhere, however the proof remains essentially the same.

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1. Barlow, R.E. and Proschan, F. (1981). Statistical Theory of Reliability and Life Testing. To Begin With, Silver Spring, MD.

2. Klefsjo, B. (1985). On Closure under Formation of Parallel Systems. IEEE Transactions on Reliability. R-33(5), 423.

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